Course Code : BCS-054
Course Title : Computer Oriented Numerical Techniques
Assignment Number : BCA(V)/054/Assignment/2015
Maximum Marks : 100
Weightage : 25%
Last Dates for Submission : 15th October, 2015 (For July 2015 Session)
15th April, 2016 (For January 2016 Session)

1.
(a) Explain each of the following concepts, along with at least one suitable example for each:
(i) Fixed-point number representation (ii) round-off error (iii) representation of zero as floating point number (iv) significant digits in a decimal number representation (v) normalized representation of a floating point number (vi) overflow

Ans. (i)
Definition 1: A number ξ is called a fixed point of g(x) if g(ξ) = ξ and g is called the iteration function. Our problem is now to find out fixed point(s) of g(x). Graphically x = g(x) is equivalent to solving y = x and y = g(x). Once an iteration function is chosen, to solve x = g(x), we start with some suitable value x_0 close to the root (how to choose this will be explained) and calculate
$x_1 = g(x_0)$ (the first approximation), then $x_2 = g(x_1)$ (second approximation) and so on.
In general
$x_{n+1} = g(x_n)$, n = 0, 1, 2 …
The sequence $\{x_n\}$ converges (under some suitable conditions on g) to a number ξ (say). If g is continuous then this gives ξ = g(ξ), that is, ξ is a fixed point of g(x).

(ii)

Rounding-off error :- There are two ways of translating a given real number x into floting-point number f(x)- routing and chopping. for example, suppose we want to represent the number 5562 in the normalized floating point representation.

(iii) Definition 1 (Floating Point Numbers): Scientific calculations are usually carried out in floating point arithmetic in computers.

An n-digit floating-point number in base β (a given natural number), has the form

$x = \pm (.d_1 d_2 \ldots d_n)_\beta^{e}$, $0 \leq d_i < \beta$, $m \leq e \leq M$; $I = 1, 2, \ldots n$, $d_1 \neq 0$; β

where $(.d_1 d_2 \ldots d_n)_\beta$ is a β– fraction called mantissa and its value is given by

$(.d_1 d_2 \ldots d_n)_\beta = d_1 \times \beta 1 + d_2 \times 21\beta + + d_n \times n\beta 1$; e is an integer called the exponent.

The exponent e is also limited to range $m < e < M$, where m and M are integers varying from computer to computer. Usually, $m = -M$.

In IBM 1130, m = –128 (in binary), –39 (decimal) and M = 127 (in binary), 38 (in decimal).

For most of the computers β = 2 (binary), on some computers β = 16 (hexadecimal) and in pocket calculators β = 10 (decimal).

The precision or length n of floating-point numbers on any computer is usually determined by the word length of the computer.

(iv)

(v) Significant digits in a decimal representation:-

The concept of significant digits has been introduced primarily to indicate the accuracy of a numerical value. For example, if in the number y = 23.40657, only the digits 23406 are correct, then we may say that y has given significant digits and is correct to only three decimal places.

(v)

(i) normalized representation of a floating point number :-

Normalized form of floating number representation is that we can have more number that can be represented exactly, instead of just approximately. And, also whenever, a number cannot be represented exactly in normalized form, then, at least, the normalized form gives better approximation of the number than can be given by any of the corresponding un-normalized form.

(vi)

(iv) Overflow :- The error, due to the the fact that the result cannot be stored even approximately. because the resultant number is too large, as its exponent is more than largest exponent that can be stored, is called 'overflow'. The overflow is more serious type of error than earlier discussed errors, because, in this case, the result cannot be stored even approximately.

(b) Explain with suitable example that in computer arithmatics (i.e., numbers represented in computer, with +, −, *, / as implemented in a computer) the multiplication operation(*) may not be distributive over plus , i.e. may not be true for some computer numbers a, b and c

ans.

(i) $((a*b)*c) = (a*b*c)$

$((a*b)*c) = a*(b*c)$

$((a*b)*c) = (a)*(b*c)$

$((a*b)*c) = (a*(b*c)$

by associativity of multiply of numbers by dif of multiply

(c) Find out to how many decimal places the value 22/ 7 is accurate as an approximation of 3.14159265, where the latter is value of π, calculated up to 8 places after decimal ?

ans.

Example - If p = 3.14159265, then find out to how many decimal places the approximate value of 22/7 is accurate ?

Ans. - we find that

22

0.00126449

7

P ↕ =

since, 0.00126449<,10

2

1

0.005 = 1. Hence, k = 2, and we conclude that the approximation is accurate to 2 decimal places or three significant digits.

(d) Calculate a bound for the truncation error in approximating f(x) = sin x by
sin (x) = x − x 3 / (fact 3) + x5 / (fact 5),
where −1 =< x =< 1 and (fact n) denotes factorial of n

ans.

Let $x = 0$ as $-1 <= x >= 1$

$f(x) = \sin x$

$f(0) = \sin 0 => 0$

Now

$\sin(x) = 1 - x^3/(fact\ 3) + x^5/(fact\ 5) - x^7/(fact\ 7),$

$\sin(0) = 1 - 0/(fact\ 3) + 0/(fact\ 5) - 0/(fact\ 7)$

$\rightarrow 1 - 0 + 0 - 0 => 1$

So

Truncation Error $= x - x'$

$\rightarrow 0 - 1 => -1$

4 (b)

Ans:- Backword difference (B.D) operator: ∇ (Inverted D elta)

$\delta f_{(x)} = f_{(x)} - f_{(x-h)}$

(c)

Central Difference (C.D) operator : δ (small Delta)

$\{ \delta f_{(x)} = f(x + \frac{1}{2}h) - f(x - \frac{1}{2}h) \}$